Calculating the Galois Group of $L_1(L_2(y)) = 0$, L_1, L_2 Completely Reducible Operators

ABSTRACT

P. H. Berman M.F. Singer
Department of Mathematics, Box 8205,
North Carolina State University,
Raleigh, NC 27695-8205
phberman@eos.ncsu.edu singer@math.ncsu.edu
http://www.math.ncsu.edu/~singer

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In *Calculating Galois groups of completely reducible linear operators*, Compoint and Singer describe a decision procedure that computes the Galois group of a completely reducible linear differential operator with rational or algebraic function coefficients (i.e., a linear differential operator that is the least common left multiple of irreducible operators or, equivalently, one whose Galois group is a reductive group). At present, it is unknown how to calculate the Galois group of a general operator. In this paper, we push beyond the completely reducible case by showing how to compute the Galois group of an operator of the form $L_1 \circ L_2$ where L_1 and L_2 are completely reducible and have rational function coefficients.

We begin by showing how to compute the Galois group of an equation of the form $L(y) = b$ with L completely reducible. This corresponds to the case of $L_1 \circ L_2$ where $L_1 = D - b'/b$. We then show how one can reduce the general case to the above case and give several examples.