>

Kernel Curves: Maple Calculations

1. Models whose discriminant has a double zero at (a,b), b nonzero.

```
> restart:
> K := expand(x*y*(1-t*(add(add(d[i,j]*x^i*y^j, i=-1..1), j=-1..1)))):
> DX := discrim(K, x):
> DD := discrim(discrim(K, x), y):
> ldegree(DD, t); degree(DD, t);
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$$4$$

$$12 \qquad \textbf{(1}$$

```
> S := solve({seq(coeff(DD, t, i), i=4..12)}, [seq(seq(d[i,j], i=-1..1), j=-1..1)]):
> nops(S);
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$$8 \qquad \textbf{(2}$$

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> S[1]; S[2]; S[3]; S[4]; S[5]; S[6]; S[7]; S[8];
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$$[d_{-1,-1}=0, d_{0,-1}=0, d_{1,-1}=0, d_{-1,0}=d_{-1,0}, d_{0,0}=d_{0,0}, d_{1,0}=d_{1,0}, d_{-1,1}=d_{-1,1}, d_{0,1}=d_{0,1}, d_{1,1}=d_{1,1}]$$

$$[d_{-1,-1}=0, d_{0,-1}=0, d_{1,-1}=d_{1,-1}, d_{-1,0}=0, d_{0,0}=d_{0,0}, d_{1,0}=d_{1,0}, d_{-1,1}=d_{-1,1}, d_{0,1}=d_{0,1}, d_{1,1}=d_{1,1}]$$

$$[d_{-1,-1}=0, d_{0,-1}=d_{0,-1}, d_{1,-1}=d_{1,-1}, d_{-1,0}=0, d_{0,0}=d_{0,0}, d_{1,0}=d_{1,0}, d_{-1,1}=0, d_{0,1}=d_{0,1}, d_{1,1}=d_{1,1}]$$

$$[d_{-1,-1}=d_{-1,-1}, d_{0,-1}=0, d_{1,-1}=0, d_{-1,0}=d_{-1,0}, d_{0,0}=d_{0,0}, d_{1,0}=0, d_{-1,1}=d_{-1,1}, d_{0,1}=d_{0,1}, d_{1,1}=d_{1,1}]$$

$$[d_{-1,-1}=d_{-1,-1}, d_{0,-1}=d_{0,-1}, d_{1,-1}=0, d_{-1,0}=d_{-1,0}, d_{0,0}=d_{0,0}, d_{1,0}=0, d_{-1,1}=d_{-1,1}, d_{0,1}=d_{0,1}, d_{1,1}=0]$$

$$[d_{-1,-1}=d_{-1,-1}, d_{0,-1}=d_{0,-1}, d_{1,-1}=d_{1,-1}, d_{-1,0}=0, d_{0,0}=d_{0,0}, d_{1,0}=d_{1,0}, d_{-1,1}=0, d_{0,1}=0, d_{1,1}=d_{1,1}]$$

$$[d_{-1,-1}=d_{-1,-1}, d_{0,-1}=d_{0,-1}, d_{1,-1}=d_{1,-1}, d_{-1,0}=d_{-1,0}, d_{0,0}=d_{0,0}, d_{1,0}=d_{1,0}, d_{-1,1}=0, d_{0,1}=0, d_{1,1}=0]$$

$$[d_{-1,-1}=d_{-1,-1}, d_{0,-1}=d_{0,-1}, d_{1,-1}=d_{1,-1}, d_{-1,0}=d_{-1,0}, d_{0,0}=d_{0,0}, d_{1,0}=0, d_{-1,1}=d_{-1,1}, d_{0,1}=0, d_{1,1}=0] \qquad \textbf{(3}$$

2.Models whose discriminant has a double zero at (a,b), b nonzero (using the prime decompostion of the radical of the ideal).

```
> with(PolynomialIdeals):
> J := <seq(coeff(DD, t, i), i = 4..12)>:
> PrimeDecomposition(J);
```

$$\langle d_{-1,-1}, d_{-1,0}, d_{-1,1}\rangle, \langle d_{-1,-1}, d_{-1,0}, d_{0,-1}\rangle, \langle d_{-1,-1}, d_{0,-1}, d_{1,-1}\rangle, \langle d_{-1,0}, d_{-1,1}, d_{0,1}\rangle, \langle d_{-1,1}, d_{0,1}, d_{1,1}\rangle, \langle d_{0,-1}, d_{1,-1}, d_{1,0}\rangle, \langle d_{0,1}, d_{1,0}, d_{1,1}\rangle, \langle d_{1,-1}, d_{1,0}, d_{1,1}\rangle \quad \textbf{(4}$$

3. Models whose discriminant has a double zero at (a,0).

```
> DDX := expand(z^4 subs(y = 1/z, DX)):
> coeff(DDX, z, 0); coeff(DDX, z, 1);
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$$-4\,t^2\,d_{-1,1}\,d_{1,1} + t^2\,d_{0,1}^2$$

$$-4\,t^2\,d_{-1,0}\,d_{1,1} - 4\,t^2\,d_{-1,1}\,d_{1,0} + 2\,t^2\,d_{0,0}\,d_{0,1} - 2\,t\,d_{0,1} \quad \textbf{(5}$$

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